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Inflation in 6D gauged supergravity

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Abstract: In this note we demonstrate that chaotic inflation can naturally be realized in the context of an anomaly free minimal gauged supergravity in $D = 6$ which has recently been the focus of some attention. This particular model has a unique maximally symmetric ground state solution, $R^{3,1} \times S^2$ which leaves half of the six-dimensional supersymmetries unbroken. In this model, the inflaton field ϕ originates from the complex scalar fields in the $D = 6$ scalar hyper-multiplet. The mass and the self couplings of the scalar field are dictated by the $D = 6$ lagrangian. The scalar potential has an absolute minimum at $\phi = 0$ with no undetermined moduli fields. Imposing a mild bound on the radius of S^2 enables us to obtain chaotic inflation. The low energy equations of motion are shown to be consistent for the range of scalar field values relevant for inflation.

Keywords: Supergravity Models, Cosmology of Theories beyond the SM.

JHEP10(2004)022

Contents

1. Introduction

Cosmological inflation is the most widely accepted solution [1] to the famous cosmological problems of standard big bang cosmology, and it provides a compelling mechanism for the origin of cosmological fluctuations (see e.g. [2] for a comprehensive review). Despite its successes, we still lack a compelling model in which inflation can be realized without unnatural fine tunings. The origin of the scalar field which triggers inflation and the range of parameters in its potential do not have a natural explanation. Whether or not there are fundamental scalar particles in nature is a question which will hopefully be answered by experiments. Superstring theories and their low energy supergravities compactified to lower than 10 space-time dimensions normally contain a plethora of scalar fields with a potential whose structure is tightly constrained by supersymmetry. The difficulty with such models is that they have Anti-de-Sitter space as their most symmetric solution, that the Calabi-Yau compactifications of 10 dimensional superstrings lack in uniqueness, and that it is not clear what mechanism will prefer and select such complicated manifolds from simpler flat 10 dimensional space-times.

The situation is different in $N = 1$ gauged supergravities (also known as (1,0) type models). In $D = 6$, such theories have the minimum number of possible supersymmetries, namely 8 real supercharges. By construction, they are chiral and therefore they can give rise to interesting low energy chiral effective gauge theories in $D = 4$. But perhaps more interestingly, they do not admit the flat 6 dimensional space-time as a solution. In fact, it has been shown recently [3] that their unique maximally symmetric ground state solution with a compact smooth 2-dimensional internal manifold is nothing but $R^{1,3} \times S^2$. There are solutions of this type which also preserve half of the $D = 6$ supersymmetries [4]. The low energy $D = 4$ theory is thus a $N = 1$ supergravity model.

These models being chiral are potentially inconsistent unless the chiral anomaly cancels. So far only one anomaly free model of this type is known [5]. This model will be the framework for our analysis and, to the extent that we need for our present discussion, it will be explained in section 2.

JHEP10(2004)022

In this paper, we show that the scalar potential of the particular class of models which we shall examine has the correct structure to generate chaotic inflation. Inflation is triggered by complex scalar fields coming from hyper-matter multiplets. The parameters of the potential are fixed completely by supersymmetry. We shall show that, provided a lower bound on the radius of compactification is satisfied, we can obtain an acceptable mass and acceptable self couplings of the scalar field which can give rise to viable inflationary cosmology of chaotic inflation type [6].

One of the distinguishing features of the supergravity models that we consider is that they have a global SU(2) R-symmetry. One can gauge either the entire SU(2) or a U(1) subgroup of it. In the following we shall consider a specific model in which a U(1) subgroup of the SU(2) is gauged. We shall denote this U(1) as $\mathrm{U}(1)_R$. Our specific choice is dictated by anomaly cancellation in $D = 6$. So far only one anomaly free model of this type has been constructed and this is the model that we shall use as our prototype example, although what we shall say will be applicable to a much larger class of gauged (1,0) models in $D = 6$.

2. The model

The specific model which we shall study belongs to the class of gauged $N = 1$ supergravity models. The lagrangians for such models have been constructed in [7, 8]. Being chiral, they are potentially inconsistent due to the presence of gauge, gravity and mixed anomalies. In [5], an anomaly free model with a gauge group $E_6 \times E_7 \times \mathrm{U}(1)_R$ has been constructed which involves a hyper-scalar multiplet transforming in a 912-dimensional pseudo-real representation of E_7. So far, this model remains the only known anomaly free example of a gauged $N = 1$ supergravity in $D = 6$. The multiplet structure and a summary of the known solutions of this model together with some of its details have been given in a recent paper [9] and will not be repeated here. Another interesting and potentially relevant development (see our remarks on dilaton stablisation in section 4) is the study in [10] where the spectrum of the effective $D = 4$ theory and the low energy lagrangian of a truncated version our model has been constructed.

For our present purpose we need only to know the following facts:

(i) The bosonic sector consists of gravity, an antisymmetric tensor potential of rank two, a dilaton field σ and a multiplet of 912 complex scalar fields ϕ belonging to an irreducible representation of E_7, plus of course the vector potentials in the adjoint of the gauge group.

(ii) In a convenient parameterization, the potential for the scalars takes a very simple form

$$V = \frac{g_1^2}{\kappa^4} e^{-\kappa\sigma} \left[(1 + |\phi|^2)^2 + \frac{g_7^2}{g_1^2} (\phi T^a \phi)^2 \right] \tag{2.1}$$

where κ is the $D = 6$ gravitational coupling and has a dimension of square length. The g_1 and g_7 are the coupling constants of the $\mathrm{U}(1)_R$ and E_7, respectively, and they have the dimensions of a length. Finally, the T^a are the hermitean generators of E_7 in its 912-dimensional representation. The most important property of this potential is that it has a unique minimum at $\phi = 0$. Hence, there is no moduli problem in this model.

(iii) Although the $D = 6$ supersymmetry does not allow a bare cosmological constant in the action, the value of the potential at its minimum acts as a cosmological constant. This important fact permits us to obtain a very simple compactifying solution for which the extra two dimensions cover a S^2 and the remaining four dimensions cover the flat Minkowski space time $R^{1,3}$. To obtain this solution one or more of the U(1) gauge potentials should assume a magnetic monopole configuration on S^2. It has recently been shown in [3] that these maximally symmetric solutions are unique. The fact that the model itself forces $R^{3,1} \times S^2$ as a ground state solution rather than $R^{5,1}$ or an Anti-de-Sitter space is one of the most attractive features of this class of models. Its uniqueness makes it even more interesting. This should be compared with the non-uniqueness of Calabi-Yau compactifications of string theories in $D = 10$.

(iv) If the monopole configuration resides in the $\mathrm{U}(1)_R$ component of the gauge group the background preserves half of the $D = 6$ supersymmetries and thus the effective $D = 4$ theory will be an anomaly free $N = 1$ chiral supergravity. All other monopole embeddings break supersymmetry completely.

3. Inflation

Within the context of effective field theories originating from superstring or M theory, it has so far been difficult to obtain realistic models with a transient phase of cosmological acceleration (see e.g. [11]), and even successful models typically have a number of e-foldings of inflation which is insufficient [12]. Our $D = 6$ supergravity theory contains a lot of scalar fields with a potential which in principle should produce inflation. Since the parameters entering this potential are highly constrained by supersymmetry, we would like to see if there is an allowed range in the space of these parameters such that the effective 4-dimensional theory produces a viable inflating universe.

From the 4-dimensional point of view, the bosonic sector of the model will include gravitational modes as well as scalar fields coming from ϕ and its Kaluza-Klein (KK) modes. The KK modes will belong to various representations of the SU(2) isometry group of S^2 . The details of the scalar potential will depend on which combination of U(1) subgroups we identify with the magnetic monopole configuration. If the monopole resides in $\mathrm{U}(1)_R \times E_7$, then the laplacian acting on ϕ will have no SU(2) singlet zero mode (unless the magnetic charges coming from the $\mathrm{U}(1)_R$ and the E_7 factors cancel out precisely) [13] and the potential will mix all the non-singlet KK modes. On the other hand, if the monopole sits in the E_6 factor of the gauge group, since the $\phi' s$ are singlets with respect to this group, the laplacian will have a zero mode which can be treated independently from the higher KK modes by simply setting these modes to zero. The potential for the zero mode will simply be given by the same expression as (2.1), except that the constant term

JHEP10(2004)022

coming from 1 on the right hand side will be absent, i.e.

$$V = 4\pi a^2 \frac{g_1^2}{\kappa^4} e^{-\kappa\sigma} \left[2|\phi|^2 + |\phi|^4 + \frac{g_7^2}{g_1^2} (\phi T^a \phi)^2 \right] , \tag{3.1}$$

where a is the radius of the S^2 and the factor of $4\pi a^2$ originates from integrating over S^2. In this potential, σ denotes the zero mode of the dilaton with respect to the laplacian, and it is a SU(2) singlet.

In the case that ϕ couples to a net magnetic monopole of charge n, its lowest mode will be in a (2n + 1)-dimensional representation of SU(2) [13] and therefore it should carry appropriate SU(2) indices which we suppress.

Here we shall consider only the lowest lying modes of the laplacian acting on the $\phi' s$ and will set to zero all the higher modes. The potential for such modes will have the general structure given above. It is not hard to work out its details and write the expressions exactly as it has been done for some examples in [14]. We shall not need such details for our discussion so we will restrict our attention to the formula (3.1).

The background solution establishes a relationship between κ, g_1, the constant vacuum expectation value σ of the dilaton, and the radius a of S^2, which is

$$a^2 = 2\frac{\kappa^2}{g_1^2} e^{\kappa\sigma} . \tag{3.2}$$

Note also that given κ, the radius a and the four-dimensional Planck mass m_{pl} are related via

$$m_{pl}^2 = 4\pi a^2 \kappa^{-2} . \tag{3.3}$$

Using the relation (3.2), and also redefining ϕ so that it has a correct mass dimension of 1 in $D = 4$ (we replace ϕ in (3.1) by ϕ/m_{pl}) one can show that the masses m_ϕ of the lowest lying ϕ modes will be (modulo a constant of the order one)

$$m_\phi^2 = \frac{1}{a^2} . \tag{3.4}$$

The scalar potential for these modes involves two quartic self couplings which are (making use of (3.3), and again modulo a constant of the order one)

$$\lambda_1 = \frac{\kappa^2}{a^4} \qquad \text{and} \qquad \lambda_2 = \frac{g_7^2}{g_1^2}\frac{\kappa^2}{a^4} . \tag{3.5}$$

The potential (3.1) allows for chaotic inflation. For a single scalar field, inflation takes place while the value of the rescaled field ϕ satisfies $|\phi| > m_{pl}$ (it is for such field values that the slow-roll approximation for the scalar field evolution is self-consistent, see e.g. [15] for a recent review). As first realized in [16] and recently discussed in more detail in [17], the slow-rolling approximation is self-consistent for an even larger range of field values if the inflaton is a multiplet of $N \gg 1$ equivalent scalar fields, as in our case. To see this, let us for simplicity drop the quartic coupling terms, and take the absolute values of all N scalar fields to be comparable. In this case, it follows from the Klein-Gordon equation for the i'th scalar field ϕ_i

$$\ddot{\phi}_i + 3H\dot{\phi}_i = -m_\phi^2 \phi_i , \tag{3.6}$$

coupled with the Friedman equation relating the Hubble expansion rate H with the potential energy, that the velocity $\dot{\phi}_i$ is suppressed by a factor of $N^{-1/2}$ compared to the result for single field inflation, and that hence the dominance of the energy density by potential energy, one of the two slow-roll conditions, remains valid for field values $|\phi|$ a factor of $N^{1/2}$ smaller than for single field inflation. In our case, since $N = 1824$, this is an important effect. In particular, it implies that the stage of inflation relevant for forming the observed large-scale structure of the Universe corresponds to field values smaller than the Planck scale, and thus corrections to the potential from non-renormalizable terms in the supergravity action are negligible (see the discussion in the following section). Given the values of the masses and coupling constants of (3.4) and (3.5), (and assuming that g_1 and g_7 are of the same order of magnitude) it also implies that the mass and quartic terms have comparable magnitudes for field values relevant to the final 50 e-foldings of inflation.

In order for quantum vacuum fluctuations produced during the period of inflation not to produce too large an amplitude of the spectrum of cosmic microwave anisotropies [18], one must have (see e.g. [15])

$$(\frac{m_\phi}{m_{pl}})^2 \leq 10^{-12} \tag{3.7}$$

or

$$\lambda_i \leq 10^{-12}\,, \tag{3.8}$$

depending on which term in the potential dominates the scalar field dynamics at the time when scales of cosmological interest today exit the Hubble radius during inflation. In our case, both conditions reduce to the same fairly mild condition on the radius of the internal S^2 manifold, namely $am_{pl} \geq 10^6$ or

$$\frac{a}{\kappa^{1/2}} \geq 10^3\,. \tag{3.9}$$

Thus, the requirement is that the radius of S^2 is larger than the 6-D gravitational length $\kappa^{1/2}$ by a factor of 10^3.

4. Discussion

In this note we have studied one aspect of early Universe cosmology in the only known anomaly-free, gauged (1,0) supergravity in six dimensions, a model in which the unique vacuum state is Minkowski space-time cross an internal S^2. We have shown that inflationary dynamics of chaotic type ("large-field" inflation) consistent with the cosmological constraints coming from the amplitude of cosmic microwave anisotropies can be realized provided that the radius of the internal space S^2 satisfies the constraint (3.9). Since the inflaton is a multiplet of $N = 1824$ scalar fields, slow-roll inflation is consistent for much smaller field values than would be the case for a single inflaton field, namely for field values satisfying $\phi < N^{-1/2}m_{pl}$. It is important to check that the potential used is consistent for such field values. The breakdown of the validity of the renormalizable potential at field values relevant for inflation is often a problem for models of chaotic inflation. A necessary condition for the applicability of the potential (3.1) is that the potential energy density during inflation is smaller than the four-dimensional Planck density. This condition is trivially satisfied given the constraints (3.7) and (3.8). Since our model comes from a higher dimensional theory, a more reasonable (and more restrictive) condition is to demand that the potential energy density during inflation is less than the six-dimensional Planck energy density κ^{-2}. For field values $\phi \sim m_{pl}$ this condition would only marginally be satisfied. However, the fact that $N \gg 1$ alleviates this potential problem, since for field values $\phi \sim N^{-1/2} m_{pl}$ we have

$$V(\phi) \sim \kappa^{-2} \,. \tag{4.1}$$

To obtain this result, we have assumed that all N fields have comparable magnitudes, and that hence $|\phi|^2$ is given by $N|\phi_i|^2$ and $|\phi|^4$ is given by $N^2|\phi_i|^4$, where ϕ_i is the characteristic value of a single field.

However, there may be a more stringent constraint: even field values larger than the typical gravitational energy scale could be problematic, and since the gravitational scale of relevance is the six-dimensional one, our approximations would not be under control for field values needed for inflation. However, as emphasized recently in [19], in the case of a theory in which there is a shift symmetry of the lagrangian in the absence of the potential energy term, there is no physical reason which demands the field values to be limited to sub-Planckian values, and the only constraints to be imposed are the above energetic ones. In our model, apart from the local gauge symmetries the lagrangian is also invariant under the quaternionic group Sp(456), which acts non trivially in the hypermatter sector and which is a much larger group than the shift symmetry group. The arguments of [19] should extend to theories with this larger symmetry group. Thus, we have shown that the low energy field equations which we have used are consistent for the field values which give us inflation.

In this paper we examined the possibility of chaotic inflation in the dimensionally reduced model. It will be interesting to see if the 6-dimensional field equations generate inflation of the type which the effective 4-dimensional theory can give rise to. In particular, it will be interesting to see if one can find an inflationary solution in which the radius of the extra two dimensions remain stable while our 3-dimensional world inflates. It has been shown a long time ago that the quantum free energy of a real scalar field can act as a source of a radiation dominated universe of this kind [21], see also the recent paper [22]. Whether an inflationary solution of this kind exists is not known yet.

This stability of the radius of the S^2 is intimately related to the stabilisation of dilaton. As is seen from equation (3.2) at the classical level the value of the dilaton σ is undetermined. Whether or not the quantum corrections lift the flat direction of the dilaton field in our model needs further study. The analogous problem of the stabilisation of the dilaton and other moduli in string compactifications on the Calabi-Yau spaces has attracted a lot of attention after the ground breaking work of ref. [23] and the progress made in [24]. The stringy origin of the model studied in this paper is still not well understood (see however, [25, 26] for some partial results). Also the stringy ideas may not be applicable to the 6-dimensional supergravity that we are considering, unless our model and its solutions are embedded in a $D = 10$ string theory in a well defined way. Nevertheless,

JHEP10(2004)022

there is encouraging evidence that the degeneracy may be lifted at the quantum level and by taking non-perturbative effects into account. This evidence comes from the detailed study carried out in [10] in a truncated version of our model, where only the gauge group U(1) is retained (the so called Salam-Sezgin Model). It has been shown by the authors of [10] that if one takes into account the improvements of renormalization group effects in the Kahler potential of the effective $D = 4$ theory together with the non perturbative super potential for the dilaton then the problem of the run-away dilaton may be avoided and, at least for some values of the parameters, new minima at finite values of σ can be created. It will be interesting to extend the analysis of [10] to the full fledged anomaly free model which is the framework of the present work.

Note that this anomaly-free $D = 6$ supergravity model admits an easy realization of the gravitational baryogenesis mechanism recently suggested in [27]. This mechanism makes use of the gravitational anomaly [28] to relate the divergence of the baryon current to $\tilde{R}R$, where R stands for the Riemann tensor. This source vanishes for a homogeneous and isotropic metric, but it is non-vanishing in the presence of a spectrum of gravitational waves, provided there is a source of cosmological bi-refringence. This birefringence is in our model generated by certain components of a second rank antisymmetric tensor potential which is necessarily present in models of the type we are considering. Details will be presented in a subsequent publication. In conclusion, we have shown that the gauged supergravity model of [5] quite naturally leads to a period of chaotic inflation in the early Universe. Further cosmological aspects of this model will be analyzed in a followup publication.

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JHEP10(2004)022